Physics Formula (Fei.Liu@njsci.org) version 1.7.2

Kinematics

$$v = v_0 + at$$
$$\Delta x = v_0 t + 1/2 a t^2$$
$$\Delta x = v_f t - 1/2 a t^2$$
$$v^2 = v_0^2 + 2a\Delta x$$
$$\Delta x = \frac{v_0 + v}{2} t$$

Dynamics

$$\Sigma \vec{F} = m\vec{a}$$
$$f_k = \mu_k N \quad f_s \leq \mu_s N$$
$$a_c = a_r = \frac{v^2}{r} = \omega^2 r$$
$$v_{\min} = \sqrt{gR}$$
$$F_g = G\frac{Mm}{r^2}$$
$$g = \frac{GM}{r^2}$$
$$U = -\frac{GMm}{r}$$
$$T = \frac{2\pi r}{v}$$
$$v_a r_a = v_p r_p$$
$$T^2/R^3 = \text{constant}$$
$$\vec{p} = m\,\vec{v}$$
$$\vec{j} = \vec{F}\,t$$
$$\vec{j} = \Delta\vec{p}$$
$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$
$$KE = 1/2\, mv^2$$
$$GPE = mgh$$
$$EPE = 1/2\, k\Delta x^2$$
$$\vec{F}_T = -k\Delta\vec{x}$$
$$W = F\,d\,\cos\theta = F_\parallel\, d = F\, d_\parallel$$
$$W_{\text{net}} = \Delta KE$$
$$W_{\text{net, non-conservative}} = \Delta E$$
$$= \Delta(KE + GPE + EPE)$$
$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t} = Fv\cos\theta$$
$$f = \frac{1}{T}$$
$$\omega = \frac{2\pi}{T} = 2\pi f$$
$$v_t \equiv r\omega \quad a_t \equiv r\alpha$$
$$v_{center} = r\omega \quad a_{center} = r\alpha \quad [r.w.s.]$$
$$\tau = rF\sin\theta = r_\perp F = rF_\perp$$
$$I_d = I_{cm} + md^2$$
$$\Sigma\vec{\tau} = I\vec{\alpha}$$
$$KE_{\text{total}} \equiv \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$
$$L_{\text{point}} = r\;mv\;\sin\theta$$
$$L_{\text{fixed\_pivot}} = I\;\omega$$
$$L_{\text{total}} = L_o + L_s = r_{cm}\;mv_{cm}\;\sin\theta + I_{cm}\;\omega$$
$$\tau_{\text{net}} t = \Delta L$$
$$T = 2\pi\sqrt{\frac{m}{k}} \quad T = 2\pi\sqrt{\frac{L}{g}}$$
$$\rho = \frac{m}{V}$$
$$p = \frac{F}{A} \quad p = \rho g h$$
$$F_{buoy} = \rho_{\text{fluid}} V_{\text{submerged}} g$$
$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$$
$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Thermaldynamics

$$l = l_0(1 + \alpha\Delta T)$$
$$V = V_0(1 + \beta\Delta T)$$
$$N = nN_A \quad R = N_A k_B$$
$$pV = nRT = Nk_B T$$
$$\overline{KE_{cm}} = \overline{KE_t} = 3/2\; k_B\; T$$
$$\Delta E = Q + W_{\text{on system}}$$
$$W = -p\Delta V$$
$$Q = mc\Delta T \quad Q = mL$$
$$\Delta S = \frac{Q}{T}$$
$$Q = \frac{kA\Delta T\; t}{\Delta x}$$
$$Q = \epsilon\sigma T^4 A\; t$$
$$\Delta S_{\text{isolated}} \geq 0$$
$$e = \frac{W}{Q_H} \leq \frac{T_H - T_L}{T_H}$$
$$cop = \frac{Q_L}{W} \leq \frac{T_L}{T_H - T_L}$$

Wave, Sound, Optics

$$v = f\lambda \quad v = \sqrt{\frac{T}{m/L}}$$
$$\lambda_n = \frac{2L}{n}$$
$$I \propto \frac{1}{r^2}$$
$$dB = 10\;\log_{10}\frac{I}{I_0}$$
$$f_{obs} = f_{src}\frac{v_{snd} \pm v_{obs}}{v_{snd} \mp v_{src}}$$
$$n = \frac{c}{v}$$
$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$\frac{1}{f} = \frac{1}{d_{obj}} + \frac{1}{d_{img}}$$
$$m = -\frac{d_i}{d_o} = \frac{h_i}{h_o}$$
$$m\lambda = d\sin\theta$$
$$\theta \approx \frac{\lambda}{D}$$
$$I = I_0\cos^2(\theta)$$

Electromagnetism

$$F_e = \frac{kQq}{r^2}$$
$$E = \frac{kQ}{r^2}$$
$$V = \frac{kQ}{r}$$
$$PE_e = \frac{kQq}{r}$$
$$\vec{F} = q\vec{E}$$
$$\Delta V = \frac{W}{q} \quad W = \Delta U = q\Delta V$$
$$\Delta V = -Ed\cos\theta = -E_\parallel d = -Ed_\parallel$$
$$R = \frac{\rho l}{A}$$
$$R_{eq} = R_1 + R_2 + R_3 ...$$
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} ...$$
$$\Delta V = IR$$
$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$
$$C = \frac{\epsilon A}{d}$$
$$Q = CV$$
$$PE_e = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} ...$$
$$C_{eq} = C_1 + C_2 + C_3 ...$$
$$I = I_0 e^{-\frac{t}{\tau}} \quad \tau = R\,C$$
$$F = qvB\sin\theta = qvB_\perp$$
$$F = ILB\sin\theta = ILB_\perp$$
$$B = \frac{\mu_0 I}{2\pi r}$$
$$B = \mu_0 nI$$
$$\mathcal{E} = -\frac{\Delta\phi}{\Delta t} = -N\frac{\Delta(BA)}{\Delta t}$$
$$\mathcal{E} = vlB$$
$$\phi = LI \quad U = \frac{1}{2}LI^2$$
$$I = I_0 e^{-\frac{t}{\tau}} \quad \tau = \frac{L}{R}$$
$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

Modern Physics

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$t = \gamma\; t_0 \quad l = \frac{l_0}{\gamma}$$
$$p = \gamma m_0 v$$
$$E = \gamma m_0 c^2 = mc^2$$
$$E = hf$$
$$hf = KE_e + W_0$$
$$\lambda = \frac{h}{m\;v}$$
$$f = \frac{E}{h} = \frac{\sqrt{p^2c^2 + m_0^2c^4}}{h}$$
$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$
$$\Delta mv_x \cdot \Delta x \geq \frac{h}{4\pi}$$
$$\Delta mv_y \cdot \Delta y \geq \frac{h}{4\pi}$$
$$\Delta mv_z \cdot \Delta z \geq \frac{h}{4\pi}$$
$$T = e^{-2\sqrt{\frac{2m(U_0 - E)}{h^2}}L}$$
$$N = N_0 e^{-kt} = N_0 e^{-t/\tau} \quad t_{\frac{1}{2}} = \frac{\ln 2}{k}$$
$$E_n = \frac{-13.6eV}{n^2}$$
$$E_n - E_m = hf_{m->n}$$

Chemistry Formula (Fei.Liu@njsci.org) version 1.3.3

Atoms and Molecules

FC=V-(N+B/2)
$F = Q_1Q_2/r^2$
$E = -Q_1Q_2/r$
541 Seasaw, 532 T-shaped, SF_4: Seasaw, IF_5: Square-pyramidal

Stoic

Mole concept and Avogadro constant $m = nM, N = nN_A$
Density mass = density · volume
Molarity M mol/L-solution and Molality m mol/kg-solvent

Gases and Liquids

Ideal gas law $pV = nRT$ and Ideal Gas Hypothesis
Deviation from ideal gas: $(P + \frac{an^2}{V^2})(V - nb) = nRT$
Kinetic Molecular Theory, $\overline{KE} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT$
Graham's Law, $r \propto \sqrt{\frac{1}{M}}$
Partial Pressure, Dalton's law of partial pressure
$p_{\text{total}} = \sum_i$ Partial Pressure$_i$
Boiling Point definition and (Saturation) Vapor Pressure
Henry's law: at constant temperature, the amount of a gas disolved in a liquid is proportional to its partial pressure of that gas in equilibrium with the liquid. $C = k_H P_{\text{partial}}$
Raoult's law: partial vapor pressure of each component of an ideal mixture of liquids is equal to the vapor pressure of the pure component multiplied by its mole fraction in the mixture.
$p_{\text{partial}} = \chi p_{\text{pure}}$ (Also applies to the vapor pressure of volatile solvent mixed with non-volatile solute)
Collegative Properties $\Delta T = i\ m\ K_b$
Osmotic pressure $\Pi = i\frac{n}{V}RT$

Kinetics

$$r_r = -\frac{\Delta[Reactant]}{\Delta t} \quad r_p = \frac{\Delta[Product]}{\Delta t}$$

$$r = -\frac{1}{a}\frac{\Delta[A]}{\Delta t} = -\frac{1}{b}\frac{\Delta[B]}{\Delta t} = \frac{1}{c}\frac{\Delta[C]}{\Delta t} = \frac{1}{d}\frac{\Delta[D]}{\Delta t}$$

$$r = k[A]^a[B]^b$$

Zeroth order $r = -k[A]^0 = -k \to [A] = [A]_0 - kt$
First order $r = -k[A] \to [A](t) = [A]_0e^{-kt}$
Second order $r = -k[A]^2 \to 1/[A](t) = 1/[A]_0 - kt$

$$k = Ae^{-\frac{E_a}{RT}}$$

$$p = p_0e^{-\frac{\Delta H_{eq}}{R}(\frac{1}{T} - \frac{1}{T_0})}$$

Half life: $k = \frac{\ln 2}{t_{1/2}}$, $t_{1/2} = \frac{\ln 2}{k}$

Equilibrium, Solution, Acid-Base

$$K_{eq} = \frac{[A]^aP_B^b}{[C]^cP_D^d}$$

$$K_w^o = 1.0 \times 10^{14}\ 25^oC$$

$$pOH = -\log[OH^-] \quad pH = -\log[H^+]$$

$$pK_w = pH + pOH$$

$$K_a = \frac{[H^+][A^-]}{[\text{HA}]} \quad K_b = \frac{[OH^-][B^+]}{[\text{BOH}]}$$

$$K_w = K_a * K_b \quad 14 = pK_a + pK_b$$

$$pH = pK_a + \log\frac{[A^-]}{[HA]}$$

$$pOH = pK_b + \log\frac{[B^+]}{[BOH]}$$

$$pk_a = pH_{\text{midpoint}}, pk_b = pOH_{\text{midpoint}}$$

$$K_{sp} = [B^+]^b[A^-]^a$$

Thermodynamics

$$Q = cm\Delta T \quad Q = n\Delta H = mL$$

$$\Delta E = W + Q = -P\Delta V + Q$$

$$W = W_{\text{mechnical}} + W_{\text{non-mechnical}}$$

$$H = E + PV$$

$$\Delta H = \Delta E + \Delta(PV) = Q + V\Delta P$$

$$G = H - TS$$

$$\Delta G = \Delta H - T\Delta S$$

$$\Delta H^o = \sum_{products} n\ \Delta H_f^o - \sum_{reactants} m\ \Delta H_f^o$$

$$\Delta S^o = \sum_{products} n\ S^o - \sum_{reactants} m\ S^o$$

$$\Delta G^o = \sum_{products} n\ \Delta G_f^o - \sum_{reactants} m\ \Delta G_f^o$$

$$\Delta G_{system}^o = \Delta H_{system}^o - T\Delta S_{system}^o = T\Delta S_{universe}^o$$

$$\Delta H^o = \sum_{reactants} m\ BDE - \sum_{products} n\ BDE$$

$$\Delta G = \Delta G^o + RT\ln Q = G_{system} - G_{minimum}$$

$$\Delta S \leq \frac{Q}{T}$$

if C_p is not given, then ΔH^0 and ΔS^0 can be treated as temperature independent, otherwise temperature dependent ΔH^0 needs to be computed from C_p.

ElectroChemistry

Cell Notation: Anode $|Anode^+||Cathode^+|$ Cathode

$$Q = It = nF$$

$$E_{cell}^o = E_{cathode}^o - E_{anode}^o$$

$$E_{cell}^o = \frac{RT}{nF}\ln K_{eq}$$

$$E = E^o - \frac{RT}{nF}\ln Q$$

$$\Delta G^o = -nFE^o = -RT\ln K_{eq} = \Delta H^o - T\Delta S^o$$

$$\Delta G = -nFE = \Delta G^o + RT\ln Q = \Delta H - T\Delta S$$

Strong Acids and Bases

Strength of Acid: binary Acid, oxyacids depends on electronegativity, polarity of bond
Strong Acids: HCl, HBr, HI, $HClO_3$, $HClO_4$, HNO_3, HSO_4, $HMnO_4$
Strong Bases: Alkali(OH), $Ca(OH)_2$, $Ba(OH)_2$..

Solubility Rules

1. All common salts of Group 1 elements and ammonium (NH_4^+) are soluble.
2. All common nitrates (NO_3^-) and acetates are soluble.
3. Most chlorides, bromides, and iodides are soluble except silver, lead (II), and mercury (I, II)
4. All sulfates are soluble except barium, strontium, lead (II), calcium, silver, and mercury (I)
5. Except for those in Rule 1, carbonates, hydroxides, oxides, sulfide, and phosphates are insoluble.

Metal Activity Series

Li, K, Sr, Ca, Na, Mg, Al, Zn, Cr, Fe, Cd, Co, Ni, Sn, Pb, H, Sb, As, Bi, Cu, Hg, Ag, Pt, Au

Pre-Calculus Reference Sheet

Factoring

- $a^2 - b^2 = (a-b)(a+b)$
- $a^2 + b^2$ is prime
- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^2 - 2ab + b^2 = (a-b)^2$
- $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$
- $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

Analytic Geometry

- slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- equation of a line: $y - y_1 = m\,(x - x_1)$
- distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Exponent Rules

- $a^{x+y} = a^x a^y$
- $(ab)^x = a^x b^x$
- $(a^x)^y = a^{xy}$
- $a^0 = 1$ if $a \neq 0$
- $a^{-x} = \dfrac{1}{a^x}$ if $a \neq 0$
- $a^{x-y} = \dfrac{a^x}{a^y}$ if $a \neq 0$

Logarithm Rules

- $\log_b x = y \iff x = b^y$
- $b^{\log_b x} = x$
- $\log_b b^x = x$
- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b xy = \log_b x + \log_b y$
- $\log_b \dfrac{x}{y} = \log_b x - \log_b y$
- $\log_b x^y = y \log_b x$
- $\log_b x = \dfrac{\log_a x}{\log_a b}$

Arithmetic Series

- $a_k = a + (k-1)d$
- $S_n = \sum_{k=1}^{n} [a + (k-1)d] = \dfrac{n}{2}\,[2a + (n-1)d]$
- $S_n = \sum_{k=1}^{n} [a + (k-1)d] = n\left(\dfrac{a + a_n}{2}\right)$

Geometric Series

- $a_n = ar^{n-1}$
- $S_n = \sum_{k=0}^{n-1} ar^k = a\left[\dfrac{1 - r^n}{1 - r}\right]$ if $r \neq 1$
- $S = \sum_{k=0}^{\infty} ar^k = \dfrac{a}{1 - r}$ if $|r| < 1$

Trigonometry

- $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
- $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$
- $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

	$0°$	$30°$	$45°$	$60°$	$90°$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Pythagorean Identities

- $\cos^2 A + \sin^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \csc^2 A$

Ratio Identities

- $\tan A = \dfrac{\sin A}{\cos A}$
- $\cot A = \dfrac{\cos A}{\sin A}$

Reciprocal Identities

- $\sec A = \dfrac{1}{\cos A}$ • $\csc A = \dfrac{1}{\sin A}$ • $\cot A = \dfrac{1}{\tan A}$

Sum and Difference Identities

- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double Angle Identities

- $\cos 2A = \cos^2 A - \sin^2 A$
- $\cos 2A = 2\cos^2 A - 1$
- $\cos 2A = 1 - 2\sin^2 A$
- $\sin 2A = 2\cos A \sin A$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Half Angle Identities

- $\cos \dfrac{A}{2} = \pm\sqrt{\dfrac{1 + \cos A}{2}}$
- $\sin \dfrac{A}{2} = \pm\sqrt{\dfrac{1 - \cos A}{2}}$
- $\tan \dfrac{A}{2} = \dfrac{1 - \cos A}{\sin A} = \dfrac{\sin A}{1 + \cos A}$

Triple Angle Identities

- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\sin 3A = 3\sin A - 4\sin^3 A$

Power Reduction Identities

- $\cos^2 A = \dfrac{1 + \cos 2A}{2}$
- $\sin^2 A = \dfrac{1 - \cos 2A}{2}$
- $\tan^2 A = \dfrac{1 - \cos 2A}{1 + \cos 2A}$
- $\cos^3 A = \dfrac{3\cos A + \cos 3A}{4}$
- $\sin^3 A = \dfrac{3\sin A - \sin 3A}{4}$

Sum-to-Product Identities

- $\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
- $\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$
- $\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
- $\cos A - \cos B = -2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$

Product-to-Sum Identities

- $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$
- $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$
- $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

Sums of Sines and Cosines

- $A\cos x + B\sin x = \sqrt{A^2 + B^2}\sin(x + \phi)$ where $\cos\phi = \dfrac{B}{\sqrt{A^2+B^2}}$ and $\sin\phi = \dfrac{A}{\sqrt{A^2+B^2}}$
- $A\cos x + B\sin x = \sqrt{A^2 + B^2}\cos(x - \phi)$ where $\cos\phi = \dfrac{A}{\sqrt{A^2+B^2}}$ and $\sin\phi = \dfrac{B}{\sqrt{A^2+B^2}}$

Laws of Sines and Cosines

- $c^2 = a^2 + b^2 - 2ab\cos C$
- $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a Triangle

For a triangle with sides a, b, c and angles $\angle A$, $\angle B$, and $\angle C$,

- Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$
- Area $= \dfrac{1}{2}ab\sin C$
- Area $= \dfrac{c^2 \sin A \sin B}{2\sin C}$

Circular Section

- Arc length: $s = r\theta$
- Area: $A = \dfrac{1}{2}r^2\theta$

Calculus Formulas

Power Rules: $\frac{d}{dx}x^n = nx^{n-1}$ and $\int x^n dx = \frac{x^{n+1}}{n+1} + c$	***Product Rule:*** $\frac{d}{dx}[f(x)\cdot g(x)] = f(x)\cdot g'(x) + f'(x)\cdot g(x)$
Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$	***Reciprocal Rule:*** $\frac{d}{dx}\left[\frac{1}{g(x)}\right] = \frac{-g'(x)}{[g(x)]^2}$
Chain Rule: $\frac{d}{dx}(f \circ g)(x) = f'[g(x)]\cdot g'(x)$	***Integration-by-Parts:*** $\int u\,dv = uv - \int v\,du$

Trigonometric Functions

Derivative	***Integral***
$\frac{d}{dx}\sin x = \cos x$	$\int \sin x\,dx = -\cos x + c$
$\frac{d}{dx}\cos x = -\sin x$	$\int \cos x\,dx = \sin x + c$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \tan x\,dx = \ln\lvert\sec x\rvert + c$ $\int \sec^2 x\,dx = \tan x + c$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$ $\int \csc^2 x\,dx = -\cot x + c$
$\frac{d}{dx}\sec x = \sec x \cdot \tan x$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$ $\int \sec x \cdot \tan x\,dx = \sec x + c$
$\frac{d}{dx}\csc x = -\csc x \cdot \cot x$	$\int \csc x\,dx = \ln\lvert\csc x - \cot x\rvert + c$ $\int \csc x \cdot \cot x\,dx = -\csc x + c$

Inverse Trigonometric Functions

Derivative	***Integral***
$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-u^2}}\,dx = \sin^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	
$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$	$\int \frac{1}{a^2+u^2}\,dx = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$	
$\frac{d}{dx}\sec^{-1} x = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{1}{u\sqrt{u^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\csc^{-1} x = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	

Identities:

$$\sin^2 x + \cos^2 x = 1 \qquad \sin 2x = 2\sin x\cos x \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$1 + \cot^2 x = \csc^2 x \qquad \cos 2x = \cos^2 x - \sin^2 x \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos(x+y) = \cos x\cos y - \sin x\sin y \qquad \sin(x+y) = \sin x\cos y + \cos x\sin y$$

Exponential Functions

Derivative	***Integral***
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + c$
$\frac{d}{dx}(b^x) = (\ln b)b^x$	$\int b^x\,dx = \frac{b^x}{\ln b} + c$

Logarithmic Functions

Derivative	***Integral***
$\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}(\log_b\lvert x\rvert) = \frac{1}{(\ln b)x}$	

Definition of Log base b: $\log_b N = x \Leftrightarrow b^x = N$

Change of Base Formula: $\log_b x = \frac{\ln x}{\ln b} = \frac{\log x}{\log b}$

Identities:

$$\ln(e^x) = x \qquad e^{\ln x} = x \qquad \ln e = \log 10 = \log_b b = 1$$

$$\log_b(b^x) = x \qquad b^{\log_b x} = x \qquad \ln 1 = \log 1 = \log_b 1 = 0$$

Infinite Series: Definitions & Tests

1. *Series*: $$\begin{cases} \sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots \ (\textbf{\textit{Infinite Series}}) \\ s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n \ (\textbf{\textit{nth Partial Sum}}) \\ \textbf{\textit{if}} \lim_{n\to\infty} s_n = s \ \textbf{\textit{where}} \ s \in \Re \ \textbf{\textit{then}} \sum_{n=1}^{\infty} a_n = s \ (\textbf{\textit{Infinite Sum}}) \end{cases}$$

2. *Geometric Series*: $$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \ldots = \begin{cases} \frac{a}{1-r}, \textbf{\textit{if}} \ |r| < 1 \\ diverges, \textbf{\textit{if}} \ |r| \geq 1 \end{cases}$$

3. *P-Series*: $$\sum_{n=1}^{\infty} \frac{1}{n^p} \Rightarrow \begin{cases} converges, \textbf{\textit{if}} \ p > 1 \\ diverges, \textbf{\textit{if}} \ p \leq 1 \end{cases}$$ ***if p = 1, the series is called the harmonic series.***

4. *Quick Divergence Test*: Given $$\sum_{n=1}^{\infty} a_n \Rightarrow \begin{cases} \textbf{\textit{if}} \lim_{n\to\infty} a_n \neq 0, \textbf{\textit{then}} \sum_{n=1}^{\infty} a_n \ \textbf{\textit{diverges}} \\ \textbf{\textit{if}} \lim_{n\to\infty} a_n = 0, \textbf{\textit{then No Conclusion! Do another test!}} \end{cases}$$

5. *Integral Test:* Given $$\sum_{n=c}^{\infty} a_n, \ a_n > 0, a_n \ \textbf{\textit{decreasing}} \Rightarrow \begin{cases} \textbf{\textit{if}} \int_c^{\infty} a_n \, dn \ \textbf{\textit{converges then}} \sum_{n=c}^{\infty} a_n \ \textbf{\textit{converges}} \\ \textbf{\textit{if}} \int_c^{\infty} a_n \, dn \ \textbf{\textit{diverges then}} \sum_{n=c}^{\infty} a_n \ \textbf{\textit{diverges}} \end{cases}$$

6. *Ratio Test:* Given $$\sum_{n=c}^{\infty} a_n, \ a_n > 0 \Rightarrow \textbf{\textit{if}} \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = p \ \textbf{\textit{then}} \begin{cases} \sum_{n=c}^{\infty} a_n \ \textbf{\textit{converges, when}} \ p < 1, \\ \sum_{n=c}^{\infty} a_n \ \textbf{\textit{diverges, when}} \ p > 1, \\ \textbf{\textit{No Conclusion, when}} \ p = 1 \end{cases}$$

7. *Root Test*: Given $$\sum_{n=c}^{\infty} a_n, \ a_n > 0 \Rightarrow \textbf{\textit{if}} \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} (a_n)^{\frac{1}{n}} = p \ \textbf{\textit{then}} \begin{cases} \sum_{n=c}^{\infty} a_n \ \textbf{\textit{converges, when}} \ p < 1, \\ \sum_{n=c}^{\infty} a_n \ \textbf{\textit{diverges, when}} \ p > 1, \\ \textbf{\textit{No Conclusion, when}} \ p = 1 \end{cases}$$

8. *Limit Comparison Test:* $$\sum_{n=c}^{\infty} a_n \ \textbf{\textit{and}} \sum_{n=c}^{\infty} b_n, \ a_n > 0, b_n > 0 \Rightarrow \begin{cases} \textbf{\textit{if}} \lim_{n\to\infty} \frac{a_n}{b_n} = p, \ p > 0, \ p \ \textbf{\textit{finite}} \\ \textbf{\textit{then both series converge or both diverge}} \end{cases}$$

9. *Comparison Test:* $$\sum_{n=c}^{\infty} a_n \ \textbf{\textit{and}} \sum_{n=c}^{\infty} b_n, \ a_n \geq 0, b_n \geq 0, a_n \leq b_n \Rightarrow \begin{cases} \textbf{\textit{if}} \ b_n \ \textbf{\textit{converges then}} \ a_n \ \textbf{\textit{converges,}} \\ \textbf{\textit{if}} \ a_n \ \textbf{\textit{diverges then}} \ b_n \ \textbf{\textit{diverges}} \end{cases}$$

10. *Alternating Series Test*: Given $$\sum_{n=c}^{\infty} (-1)^n a_n, \ \textbf{\textit{if}} \ a_n > 0, a_{n+1} < a_n, \lim_{n\to\infty} a_n = 0, \textbf{\textit{then}} \sum_{n=c}^{\infty} (-1)^n a_n \ \textbf{\textit{converges}}$$

Special Relativity Formula (Fei.Liu@njsci.org) version 1.1.1

Notation

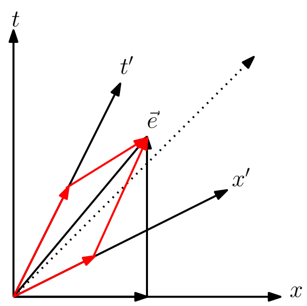
$\|\|$: magnitude
$c = 1$: speed of light
S: lab frame
S': rest (proper) frame
v: S' velocity relative to S
v': object velocity in S'
$\beta = \frac{v}{c} = v$
$\gamma = \frac{1}{\sqrt{1-\beta^2}}$
θ: rapidity
τ: proper time
m_0: rest mass
$d\vec{\boldsymbol{S}}$: displacement 4 vector
$\vec{\boldsymbol{V}}$: velocity 4 vector
$\vec{\boldsymbol{P}}$: momentum 4 vector
$\vec{\boldsymbol{F}}$: force 4 vector
L: Lorentz matrix

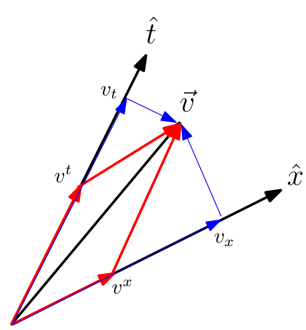
Kinematics

$$\begin{cases} dt = \gamma d\tau \\ dx = \frac{dx'}{\gamma} \\ \Delta t' = t'_{\text{rear}} - t'_{\text{front}} = \|v\Delta x'\| \end{cases} \qquad \begin{cases} dt = \gamma(dt' + vdx') \\ dx = \gamma(dx' + vdt') \end{cases}$$

$$\begin{cases} v_x = \frac{v'_x + v}{1+v'_x v} \\ v_y = \frac{v'_y}{\gamma(1+v'_x v)} \\ v_z = \frac{v'_z}{\gamma(1+v'_x v)} \end{cases} \qquad \begin{cases} \beta = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2} \\ \gamma = \gamma_1\gamma_2(1+\beta_1\beta_2) \\ \gamma\beta = \gamma_1\gamma_2(\beta_1+\beta_2) \end{cases}$$

$$\begin{cases} f_{\text{longitudinal}} = \sqrt{\frac{1+\beta}{1-\beta}} f_0 \\ f_{\text{transverse}} = \frac{1}{\gamma(1-\cos\theta)} f_0 \end{cases}$$





4 Vector

$$d\vec{\boldsymbol{S}} = (dt, dx, dy, dz)$$

$$d\vec{\boldsymbol{S}}^2 = d\vec{\boldsymbol{S}} \cdot d\vec{\boldsymbol{S}} = dt^2 - dx^2 - dy^2 - dz^2 = c^2 d\tau^2$$

$$\vec{\boldsymbol{V}} \cdot \vec{\boldsymbol{U}} = V_1U_1 - V_2U_2 - V_3U_3 - V_4U_4$$

$$\vec{\boldsymbol{V}} = \frac{d\vec{\boldsymbol{S}}}{d\tau} = \gamma\frac{d\vec{\boldsymbol{S}}}{dt} = \gamma(1, v_x, v_y, v_z) = (\gamma, \gamma\vec{\boldsymbol{v}})$$

$$\vec{\boldsymbol{A}} = \frac{d\vec{\boldsymbol{V}}}{d\tau} = \gamma\frac{d\vec{\boldsymbol{V}}}{dt} = \gamma(\frac{d\gamma}{dt}, \frac{d\gamma v_x}{dt}, \frac{d\gamma v_y}{dt}, \frac{d\gamma v_z}{dt})$$

$$\vec{\boldsymbol{A}} = \gamma(\frac{d\gamma}{dt}, \frac{d\gamma\vec{\boldsymbol{v}}}{dt}) = (\gamma v\dot{v}, \gamma^4 v\dot{v}\vec{\boldsymbol{v}} + \gamma^2\vec{\boldsymbol{a}}) = (\gamma^4 v_x a_x, \gamma^4 a_x, \gamma^2 a_y, \gamma^2 a_z)$$

$$\vec{\boldsymbol{P}} = m_0\vec{\boldsymbol{V}} = (\gamma m_0, \gamma m_0\vec{\boldsymbol{v}}) = (E, \vec{\boldsymbol{p}})$$

$$\vec{\boldsymbol{F}} = \frac{d\vec{\boldsymbol{P}}}{d\tau} = \gamma(\frac{dE}{dt}, \vec{\boldsymbol{f}}) = m_0\vec{\boldsymbol{A}}$$

$$\vec{\boldsymbol{f}} = m_0(\gamma^3 a_x, \gamma a_y, \gamma a_z)$$

$$P^2 = \vec{\boldsymbol{P}} \cdot \vec{\boldsymbol{P}} = m_0^2, \vec{\boldsymbol{v}} = \frac{\vec{\boldsymbol{p}}}{E}, \vec{\boldsymbol{v}}_{cm} = \frac{\sum\vec{\boldsymbol{p}}}{\sum E}$$

$$\sum\vec{\boldsymbol{P}}_i = \sum\vec{\boldsymbol{P}}_f$$

$$L_x = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$L_{\text{general}} = \begin{pmatrix} \gamma & \gamma v_x & \gamma v_y & \gamma v_z \\ \gamma v_x & 1+(\gamma-1)\frac{v_x^2}{v^2} & (\gamma-1)\frac{v_x v_y}{v^2} & (\gamma-1)\frac{v_x v_z}{v^2} \\ \gamma v_y & (\gamma-1)\frac{v_y v_x}{v^2} & 1+(\gamma-1)\frac{v_y^2}{v^2} & (\gamma-1)\frac{v_y v_z}{v^2} \\ \gamma v_z & (\gamma-1)\frac{v_z v_x}{v^2} & (\gamma-1)\frac{v_z v_y}{v^2} & 1+(\gamma-1)\frac{v_z^2}{v^2} \end{pmatrix}$$

$$L_{general}(v_y = 0, v_z = 0) = L_x$$

$$\vec{\boldsymbol{V}} = L\vec{\boldsymbol{V}}', V^i = L^i_j V'^j$$

Dynamics

$$\gamma^2 v^2 + 1 = \gamma^2$$

$$\dot{\gamma} = \frac{d\gamma}{dt} = \gamma^3 v\frac{dv}{dt} = \gamma^3 va, \frac{d\gamma v}{dt} = \gamma^3\frac{dv}{dt} = \gamma^3 a$$

$$d(\gamma m_0 v) = d\gamma m_0 v + \gamma dm_0 v + \gamma m_0 dv = \gamma^3 m_0 dv + \gamma v dm_0$$

The following transformations are from proper frame S' to S

$$\vec{\boldsymbol{A}} = L\vec{\boldsymbol{A}}', \vec{\boldsymbol{F}} = L\vec{\boldsymbol{F}}'$$

$$\vec{\boldsymbol{f}} = m_0(\gamma^3 a_x, \gamma a_y), \vec{\boldsymbol{f}}'(\gamma = 1) = m_0(a'_x, a'_y)$$

$$a_x = \frac{a'_x}{\gamma^3}, a_y = \frac{a'_y}{\gamma^2}, a_z = \frac{a'_z}{\gamma^2}$$

$$f_x = f'_x, f_y = \frac{f'_y}{\gamma}, f_z = \frac{f'_z}{\gamma}$$

$$v(\tau) = \tanh(\tau), t = \frac{\sinh(a_0\tau)}{a_0}$$

$$p = \gamma m_0 v, E = \gamma m_0, E^2 - p^2 = m_0^2, \frac{p}{E} = v$$

$$E = \gamma(E' + \beta p'), p = \gamma(p' + vE')$$

$$dW = Fdx = dE, dj = Fdt = dp$$

$$v_{cm} = \frac{\sum_i \gamma_i m_i v_i}{\sum_i \gamma_i m_i}$$

$$d\cosh\theta = \sinh\theta d\theta, d\sinh\theta = \cosh\theta d\theta, \cosh^2\theta - \sinh^2\theta = 1$$

$$\tanh\theta = \beta, d\beta = \operatorname{sech}^2\theta d\theta$$

Electromagnetism

$$\vec{\boldsymbol{E}}' = -\gamma(\vec{\boldsymbol{E}} + (\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}})) - (\gamma - 1)\frac{\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{v}}}{v^2}\vec{\boldsymbol{v}}$$

$$\vec{\boldsymbol{B}}' = \gamma(\vec{\boldsymbol{B}} - \frac{1}{c^2}(\vec{\boldsymbol{v}} \times \vec{\boldsymbol{E}})) - (\gamma - 1)\frac{\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{v}}}{v^2}\vec{\boldsymbol{v}}$$

$$\frac{d(\gamma m_0 \vec{\boldsymbol{v}})}{dt} = q(\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}})$$

$$\vec{\boldsymbol{E}}'_{\parallel} = \vec{\boldsymbol{E}}_{\parallel}$$

$$\vec{\boldsymbol{B}}'_{\parallel} = \vec{\boldsymbol{B}}_{\parallel}$$

$$\vec{\boldsymbol{E}}'_{\perp} = \gamma(\vec{\boldsymbol{E}}_{\perp} + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}})$$

$$\vec{\boldsymbol{B}}'_{\perp} = \gamma(\vec{\boldsymbol{B}}_{\perp} - \frac{1}{c^2}\vec{\boldsymbol{v}} \times \vec{\boldsymbol{E}})$$

Hyperbolic trignometry

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x - e^{-x}} \quad x \neq 0$$

$$\coth x = \frac{e^x + e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{2}{e^x - e^{-x}} \quad x \neq 0$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{arcsinh} x = \ln\left(x + \sqrt{x^2 + 1}\right) \qquad x \in \mathcal{R}$$

$$\operatorname{arccosh} x = \ln\left(x + \sqrt{x^2 - 1}\right) \qquad x \in [1, \infty)$$

$$\operatorname{arctanh} x = \frac{1}{2}\ln\frac{1 + x}{1 - x} \qquad -1 < x < 1$$

$$\operatorname{arccoth} x = \frac{1}{2}\ln\frac{x + 1}{x - 1} \qquad x < -1 \cup x > 1$$

$$\operatorname{arccsch} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right) \qquad x \neq 0$$

$$\operatorname{arcsech} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right) \qquad x \in (0, 1]$$

$$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{x^2 + 1}}$$

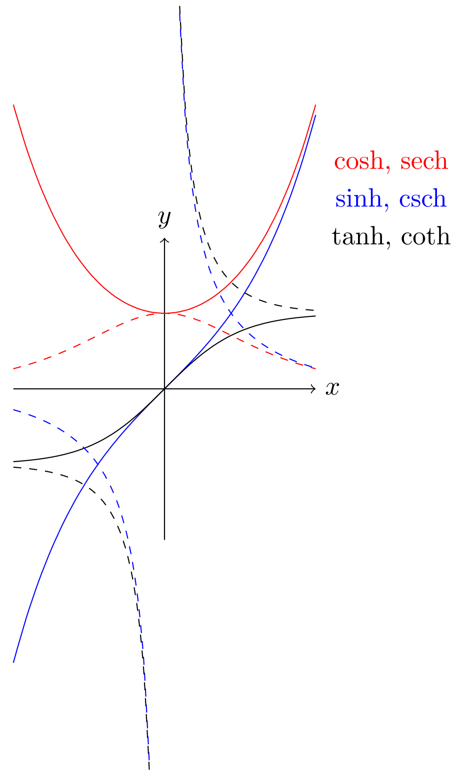
$$(\operatorname{arccosh} x)' = \frac{1}{\sqrt{x^2 - 1}}$$

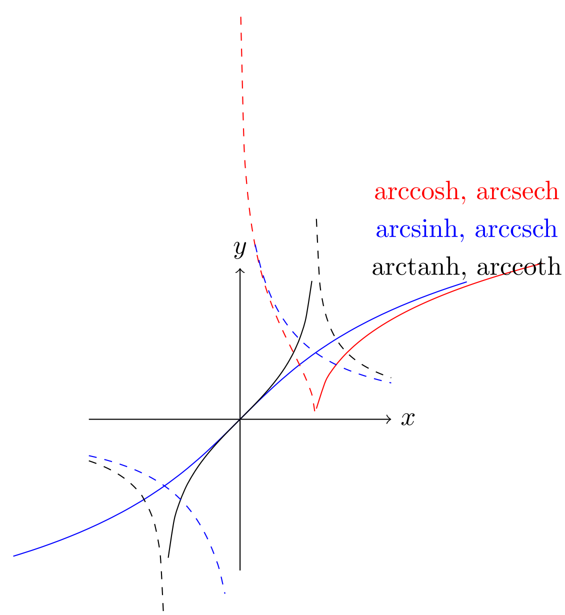
$$(\operatorname{arctanh} x)' = \frac{1}{1 - x^2} \quad |x| > 1$$

$$(\operatorname{arccoth} x)' = \frac{1}{1 - x^2} \quad |x| < 1$$

$$(\operatorname{arccsch} x)' = \frac{-1}{|x|\sqrt{1 + x^2}}$$

$$(\operatorname{arcsech} x)' = \frac{-1}{|x|\sqrt{1 - x^2}}$$





AMC10/12 Formula (Fei.Liu@njsci.org) version 1.2.6

Notation

$\wedge$: and
$\implies$: implies: if, then
$\iff$: if and only if
$\sum$: summation
$\prod$: multiplication

Algebra

Prime Factorization

$$N = p_1^n * p_2^l * p_3^m ...$$

Geometric Sequence (progression)

$$a_n = a_0 r^n, S_n = \frac{a_0(1 - r^{n+1})}{1 - r}$$

Arithmetic Sequence (progression)

$$a_n = a_0 + nr, S_n = \frac{(a_0 + a_n)(n + 1)}{2}$$

AM-GM

$$\frac{x_1 + x_2 + ... + x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdot ...x_n}$$

Mean, Median, Mode, Unique Mode, Range

$$ab + a + b = n \rightarrow (a + 1)(b + 1) = n + 1$$

Multinomial Expansion

$$(a+b+c)^n = \sum_{k_1+k_2+k_3=n} \binom{n}{k_1 \quad k_2 \quad k_3} a^{k_1} b^{k_2} c^{k_3}$$

$$\binom{n}{k_1 \quad k_2 \quad k_3} = \frac{n!}{k_1!k_2!k_3!}$$

Vieta's Formula

$$\sum r_i = -\frac{a_{n-1}}{a_n}$$

$$\sum r_i r_j = \frac{a_{n-2}}{a_n}$$

$$\prod r_i = (-1)^n \frac{a_0}{a_n}$$

Geometry

Angle Bisector Theorem

$$A = \frac{1}{2}bh = \frac{1}{2}ab\sin\theta$$
$$= \sqrt{s(s-a)(s-b)(s-c)}$$

A median of a triangle is a line segment joining a vertex to the midpoint of the opposite side, thus bisecting that side.
Cyclic Quadrilateral

$$\angle A + \angle C = \angle B + \angle D$$

Ptolemy's theorem

$$AC \cdot BD = AB \cdot CD + AD \cdot BC$$

$$AE \cdot EC = BE \cdot ED$$
$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Shoelace Theorem of polygon area

$$A = \frac{1}{2}|(x_1y_2 + x_2y3 + .. + x_ny_1)$$
$$-(y_1x_2 + y_2x_3 + .. + y_nx_1)|$$

Combinatorial

Stars and Bars I: put n indistinguishable balls into k distinguishable bins without empty bars $n > k$.

$$\binom{n-1}{k-1}$$

Stars and Bars II: put n indistinguishable balls into k distinguishable bins with empty bars $n > k$.

$$\binom{n+k-1}{k-1}$$

Stars and Bars III: put n distinguishable balls into n distinguishable bins without empty bars.

$$1 \cdot (1 - \frac{1}{n}) \cdot (1 - \frac{2}{n}) \cdot \cdot\cdot = \frac{n!}{n^n}$$

Stars and Bars III: put n distinguishable balls into n distinguishable bins with one empty bar.

$$\frac{n(n-1)n!}{2n^n}$$

Number of ways to put n_1 red balls, n_2 blue balls, n_3 green balls etc into $n_1 + n_2 + n_3 + ..$ with one ball in each box is

$$\frac{(n_1 + n_2 + n_3 + ..)!}{n_1!n_2!n_2!..}$$

This is a very useful conclusion in kinetic gas theory.

Numbers

If the last two digits of a number are divisible by 4, the number is divisible by 4.
Any even number whose digits sum to a multiple of 3 is divisible by 6.
A number is divisible by 7, then "the difference between twice the unit digit of the given number and the remaining part of the given number should be a multiple of 7 or it should be equal to 0".
If the last three digits of a number are divisible by 8, then the number is completely divisible by 8.
If the sum of digits of the number is divisible by 9, then the number itself is divisible by 9.
A number is divisible by 11 if the difference between the sum of the digits in odd places and the sum of the digits in even places is 0 or a multiple of 11.
If adding four times the last digit to the number formed by the remaining digits is divisible by 13, then the number is divisible by 13.
Number of divisors

$$n = ab, d(n) = d(a) * d(b)$$

Diophantine Equation

$$am + bn = c$$

The number of factor k in $n!$ $(k < n)$ is

$$\lfloor\frac{n}{k}\rfloor + \lfloor\frac{n}{k^2}\rfloor + \lfloor\frac{n}{k^3}\rfloor + \lfloor\frac{n}{k^4}\rfloor + ...$$

Eular Totient Function

$$\Psi(n) = n\prod_{p|n}(1 - \frac{1}{p})$$

where $p|n$ reads the prime p is a divisor of n.

Modular Arithmetic

$$a \equiv b \pmod n \iff a - b = n * m$$

Reflexivity: $a \equiv a \pmod n$
Symmetry: $a \equiv b \pmod n \iff b \equiv a \pmod n$
Transitivity: $a \equiv b \pmod n \wedge b \equiv c \pmod n \implies a \equiv c \pmod n$

$$a \equiv b \pmod n \implies a + k \equiv b + k \pmod n,$$
$$ka \equiv kb \pmod n, ka \equiv kb \pmod{kn},$$
$$a \pm a_2 \equiv b_1 \pm b_2 \pmod n, a_1a_2 \equiv b_1b_2 \quad (\text{mod}$$
$$a^k \equiv b^k \pmod n, p(a) \equiv p(b) \pmod n$$

Set

$$S_A \cup S_B = S_A + S_B - S_A \cap S_B$$

$$S_A \cup S_B \cup S_C = S_A + S_B + S_C - S_A \cap S_B - S_B \cap S_C -$$